

RPN Challenge 1 - Answer Key

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Problem 1

The largest expression is:

$2 \ 3 \ 4 \ 5 \ ^ \wedge \wedge \wedge$

This evaluates as $2^{(3^{(4^5)})}$. This seems right, but is it a proof? No. Let's consider one other possibly big number: $3 \ 2 \ 4 \ 5 \ ^ \wedge \wedge \wedge$

To compare it with $3 \ 2 \ 4 \ 5 \ ^ \wedge \wedge \wedge$, we will use logarithms. Let

$$A = 2^{(3^{(4^5)})}, \quad B = 3^{(2^{(4^5)})}.$$

Both numbers are greater than 1, so logarithms preserve their order.

Take $\ln$ of A :

$$\ln A = \ln \left(2^{(3^{(4^5)})} \right) = 3^{4^5} \ln 2.$$

Now take $\ln$ of that result:

$$\ln(\ln A) = \ln \left(3^{4^5} \ln 2 \right) = \ln \left(3^{4^5} \right) + \ln(\ln 2) = 4^5 \ln 3 + \ln(\ln 2).$$

The same two steps applied to B give:

$$\ln(\ln B) = 4^5 \ln 2 + \ln(\ln 3).$$

Now compute decimal values. We can ignore $\ln \ln 2$ and $\ln \ln 3$ because they are tiny. So we're just comparing $4^5 \ln 2$ against $4^5 \ln 3$. Obviously $\ln 3 > \ln 2$:

$$\ln(\ln A) \approx 1024(1.098612) \approx 1124.97,$$

$$\ln(\ln B) \approx 1024(0.693147) \approx 709.78.$$

Since $1124.97 > 709.78$, we have $\ln(\ln A) > \ln(\ln B)$, and since $\ln$ is increasing, this means $A > B$:

$$2\ 3\ 4\ 5\ \wedge\ \wedge\ \wedge > 3\ 2\ 4\ 5\ \wedge\ \wedge\ \wedge$$

Problem 2

The smallest expression is:

$$2\ 3\ 4\ 5\ \wedge\ \wedge\ -$$

This evaluates as $2 - 3^{(4^5)}$.

Problem 3

The closest expression to e is:

$$2\ 5\ 3\ 4\ +\ /\ +$$

This evaluates as $2 + 5/7 = 19/7 \approx 2.7142857$.

Problem 4

Using each number 2, 3, 4, 5 exactly once, and allowing the numbers to be reordered, the following RPN expressions construct each target value. The operators are +, -, *, /, and ^.

1. $2\ 3\ +\ 4\ -\ 5\ \wedge$
2. $5\ 2\ 3\ -\ 4\ +\ -$
3. $5\ 2\ 3\ *\ 4\ -\ -$
4. $2\ 3\ +\ 4\ +\ 5\ -$
5. $2\ 3\ +\ 4\ -\ 5\ *$
6. $2\ 3\ +\ 4\ -\ 5\ +$
7. $2\ 3\ *\ 4\ -\ 5\ +$
8. $2\ 3\ -\ 4\ +\ 5\ +$
9. $5\ 2\ 3\ -\ 4\ *\ -$
10. $5\ 2\ 3\ -\ 4\ -\ -$
11. $4\ 2\ 3\ /\ /\ 5\ +$
12. $2\ 4\ +\ 5\ 3\ -\ *$
13. $3\ 2\ 4\ +\ *\ 5\ -$
14. $2\ 3\ +\ 4\ +\ 5\ +$
15. $2\ 3\ +\ 4\ *\ 5\ -$
16. $3\ 2\ 4\ *\ +\ 5\ +$
17. $2\ 3\ \wedge\ 4\ +\ 5\ +$
18. $3\ 2\ \wedge\ 4\ +\ 5\ +$
19. $2\ 3\ *\ 4\ *\ 5\ -$
20. $5\ 4\ 2\ 3\ -\ \wedge\ /\$
21. $3\ 4\ +\ 5\ 2\ -\ *$
22. $3\ 4\ *\ 2\ 5\ *\ +$

23. $3 \ 2 \ 4 \ + \ * \ 5 \ +$
24. $3 \ 2 \ 4 \ ^ \ + \ 5 \ +$
25. $2 \ 3 \ + \ 4 \ * \ 5 \ +$
26. $5 \ 2 \ ^ \ 3 \ 4 \ - \ -$
27. $2 \ 3 \ ^ \ 4 \ * \ 5 \ -$
28. $4 \ 2 \ 5 \ * \ 3 \ - \ *$
29. $2 \ 3 \ * \ 4 \ * \ 5 \ +$
30. $5 \ 2 \ 3 \ / \ 4 \ / \ /$
31. $2 \ 3 \ ^ \ 4 \ * \ 5 \ -$
32. $2 \ 3 \ * \ 4 \ - \ 5 \ ^$
33. $2 \ 5 \ ^ \ 3 \ 4 \ - \ -$
34. $4 \ 2 \ 3 \ * \ 5 \ * \ +$
35. $3 \ 2 \ 4 \ - \ 5 \ ^ \ -$
36. $3 \ 4 \ * \ 5 \ 2 \ - \ *$
37. $2 \ 3 \ ^ \ 4 \ * \ 5 \ +$
38. $3 \ 4 \ ^ \ 5 \ - \ 2 \ /$
39. $3 \ 4 \ + \ 2 \ 5 \ ^ \ +$
40. $4 \ 2 \ - \ 3 \ ^ \ 5 \ *$
41. $3 \ 2 \ ^ \ 4 \ * \ 5 \ +$
42. $3 \ 4 \ 2 \ 5 \ * \ + \ *$
43. $3 \ 2 \ 4 \ ^ \ * \ 5 \ -$
44. $3 \ 4 \ + \ 2 \ ^ \ 5 \ -$
45. $2 \ 3 \ + \ 4 \ + \ 5 \ *$
46. $2 \ 3 \ 4 \ 5 \ * \ + \ *$
47. **Impossible**
48. $2 \ 4 \ + \ 3 \ 5 \ + \ *$
49. $3 \ 4 \ + \ 2 \ 5 \ + \ *$